Imperial College London

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Training coursework 3

Implementation of the stochastic Euler algorithm

We will now implement an Euler algorithm for computing the numerical solutions of the *Stochastic Differential Equation*

$$dx = -kxdt + \sigma dW, \tag{1}$$

where σ is the amplitude of the random noise process dW.

Take $k = 3/16,\ h = 0.01,\ x(0) = 6,\ t \in [0, 10]$ and $\sigma = 0.2$ and write Matlab code to solve:

$$x(t + h) = x(t) + h\,[-k\,x(t)] + \sigma\sqrt{h} * \texttt{randn},$$

where `randn` is a Matlab function that generates a random number drawn from a *Gaussian distribution* of mean zero and unit variance (see `help randn`).

1. (a) Run your code 20 times and superimpose the plots of $x(t)$ as a function of time.
   (b) Using these 20 trajectories, calculate the average trajectory as a function of time and use the mean squared error to compare it to the analytical solution obtained for the deterministic system (i.e. where $\sigma = 0$).
2. (a) Run your code with $x(0) = 0$, $\sigma = 0.1$ and $h = 0.01$ for a long time, $T \gg 10$. (Note the initial condition is zero now.) Plot the *histogram* of the values of $x(T)$.
   (b) Repeat this calculation but now with $\sigma = 5$ and plot the corresponding *histogram*.
   (c) Use Matlab to calculate the mean and standard deviation of the two distributions obtained above. Explain the difference in the width of the histograms for the two values of σ.

In this coursework you may need to use the following Matlab commands: `randn`, `function`, `plot`, `mean`, `std`, `hist`. You can check the Matlab help by using `help COMMAND`.