A second order ODE system

Consider the second order linear ordinary differential equation

\[
\frac{d^2 y}{dt^2} + \eta \frac{dy}{dt} + y = 0
\]

where \(\eta\) is a positive parameter.

1. Write the equation as a system of two first order ODEs and integrate it numerically with Matlab’s \texttt{ode45}, with initial conditions \(\{y(0) = 2, \; \dot{y}(0) = 10\}\) and \(t \in [0, 100]\). Perform the calculation for three different dampings: \(\eta = \{0, 0.03, 7\}\). Plot the three trajectories \(y(t)\) as a function of time on the same figure.

2. Represent the same trajectories in phase space by plotting \(\dot{y}\) as a function of \(y\) for the three different values of \(\eta\).

3. Explain the difference between the cases with \(\eta = 0.03\) and \(\eta = 7\) in the phase plane. At what value of \(\eta\) does one expect to switch from one behaviour to the other?

In this coursework you may need to use the following Matlab commands: \texttt{ode45}, \texttt{function}, \texttt{plot}, \texttt{hold}, \texttt{dsolve}. You can check the Matlab help by using \texttt{help COMMAND}.