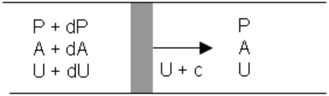
Water hammer equations

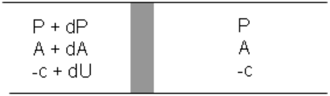
The water hammer equations follow simply from the Riemann variables in the method of characteristics solution for flow in an elastic tube. They can also be derived using an alternative, more physical approach which is presented here.

Consider a wavefront propagating into an elastic tube with pressure P, area A and velocity U. It will travel with the speed $U \pm c$ where c is the wave speed. Transforming into coordinates moving with the wavefront, we obtain the conditions shown to the right in the figure. In these coordinates the flow is steady.

tube coordinates



wavefront coordinates



Water hammer equations 2

Mass and momentum conservation require

$$(\mp c + dU)(A + dA) - (\mp cA) = 0$$

$$\rho(\mp c + dU)^2(A + dA) - \rho(\mp c)^2 A = PA - (P + dP)A$$

Evaluating these equations and neglecting terms of second order in the differences, we obtain

$$cdA = \pm AdU$$

$$\rho c(cdA \mp 2AdU) = -AdP$$

Combining, we obtain the water hammer equations

$$dP = \pm \rho c dU$$

where '+' refers to the forward and '-' to the backward travelling wavefront.