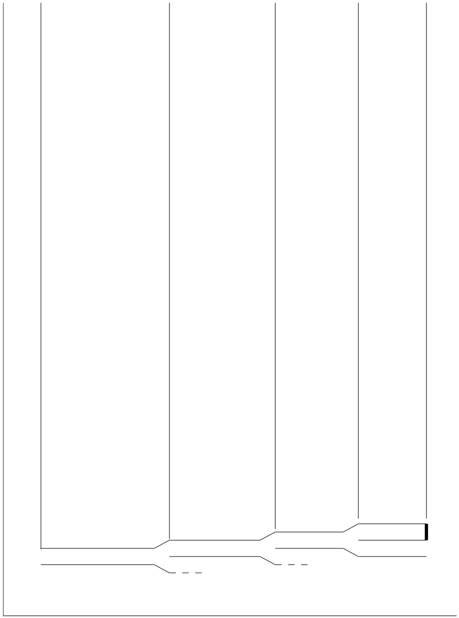
wave trapping 01

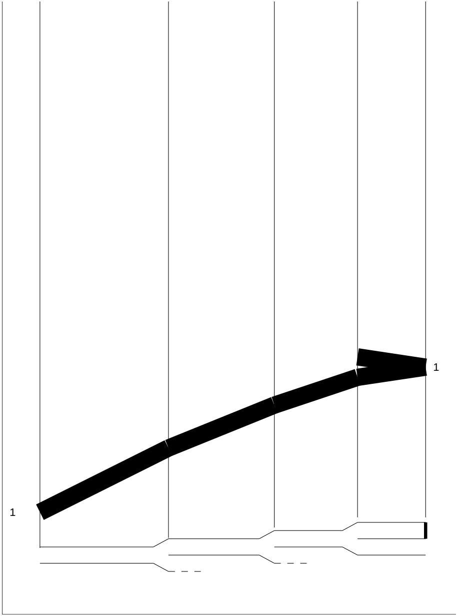
Consider a wavefront propagating forward into the symmetrical tree. Assume that the bifurcations are well-matched in the forward direction and that the reflection coefficient for backward wavefronts is $R = -\frac{1}{2}$, a realistic value for a symmetrical bifurcation. Assume that one of the vessels in the last generation is completely occluded while the others are open.

The figure shows the tree at the bottom and the x coordinate will be along the tree and the t coordinate will be the ordinate. The paths taken by the wavefronts are indicated by lines in the $x - t$ plane, the characteristic directions. A wavefront is introduced into the root of the tree with magnitude 1. We will follow it and the transmitted and reflected waves that are produced at the various bifurcations.



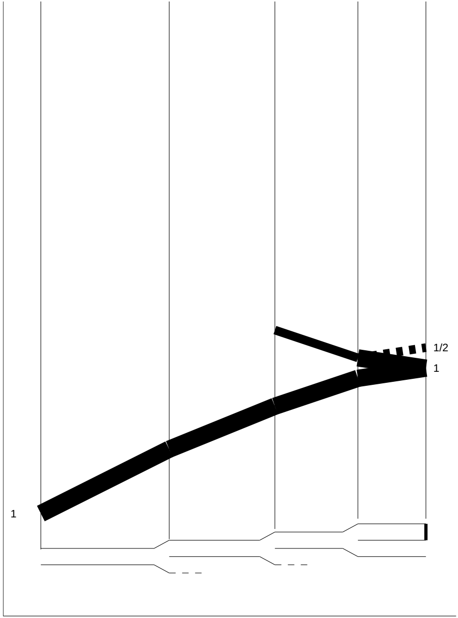
wave trapping 02

The forward compression wavefront begins at the root of the tree with magnitude 1. Because the bifurcations are well-matched, it travels without change through all of the vessels of the tree.
When it reaches the occluded end of the 4^{th} generation vessel, it is reflected with $R = 1$. This backward compression wavefront propagates back to the 3^{rd} bifurcation.



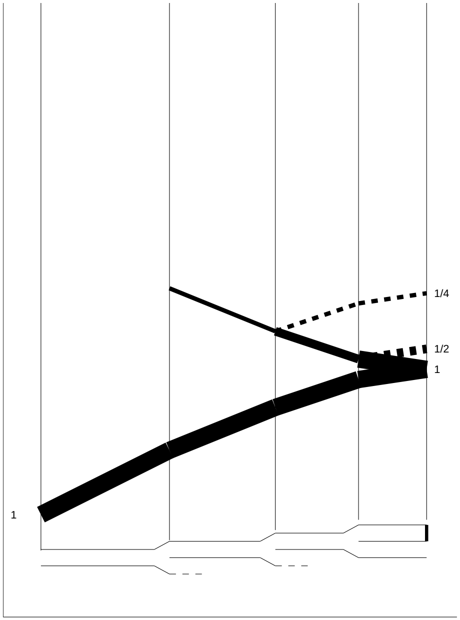
wave trapping 03

At the 3^{rd} bifurcation the reflection coefficient is $R = -1/2$, so the backward compression wavefront generates a reflected decompression wavefront of magnitude $1/2$. The transmission coefficient is $T = 1 + R = 1/2$, so a backward compression wavefront of magnitude $1/2$ is generated in the 3^{rd} generation vessel.



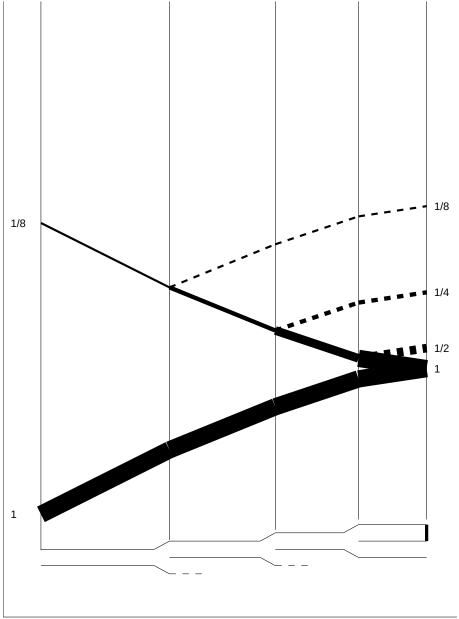
wave trapping 04

The backward compression wavefront in the 3^{rd} generation vessel is reflected when it arrives at the 2^{nd} bifurcation with $R = -1/2$, producing a forward decompression wave of magnitude $1/4$. This wave propagates forward without change until it encounters the occlusion.
$T = 1/2$ so the transmitted wave travels backward in the 2^{nd} generation vessel with magnitude $1/4$.



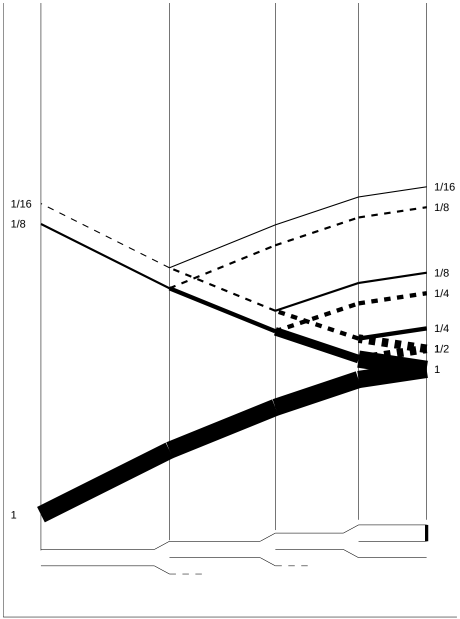
wave trapping 05

The backward compression wavefront with magnitude 1/4 is similarly reflected and transmitted when it reaches the 1^{st} bifurcation. The reflected wavefront is a decompression wave of magnitude 1/8 and it propagates forward without change until it reaches the occlusion. The transmitted wave propagates backward to the root of the tree. The first result of the occlusion at the root of the tree is therefore the arrival of a compression wavefront of magnitude 1/8.



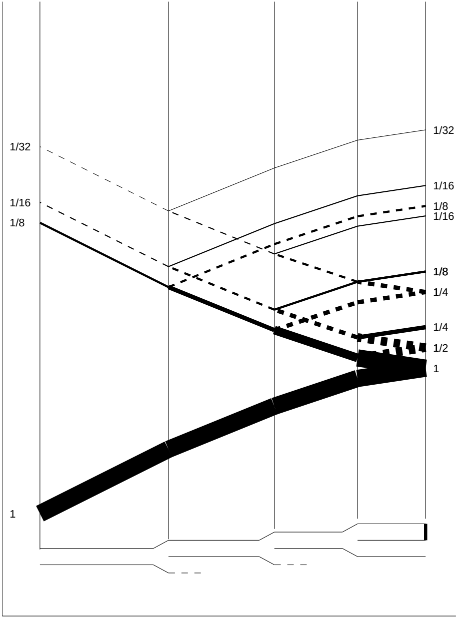
wave trapping 06

The reflected forward decompression wavefront of magnitude 1/4 that arrived at the occlusion will generate a sequence of wavefronts exactly like the original wavefront of magnitude 1, except that the magnitude of each wavefront will be reduced by 1/2 and the sign of the wavefronts will be inverted. I.e. compression wavefronts will be decompression wavefronts and decompression wavefronts will be compression wavefronts.
This will result in a decompression wavefront arriving at the root of the tree with magnitude 1/16. It will also generate 3 more forward wavefronts that will propagate to the occlusion without change.



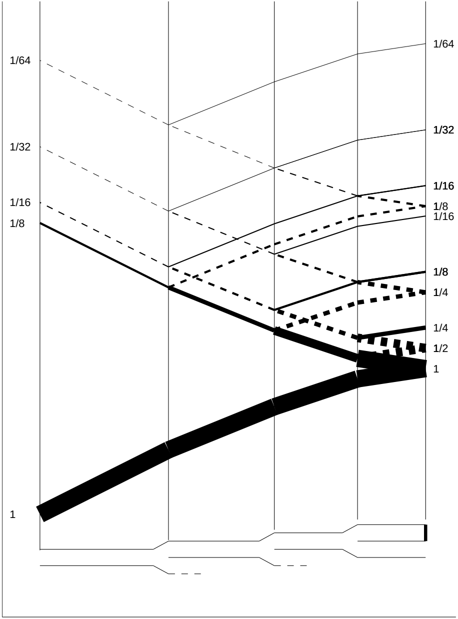
wave trapping 07

In exactly the same way, the decompression wavefront of magnitude 1/8 arriving at the occlusion from the reflection at the 2^{nd} bifurcation will generate a family of wavefronts just like the previous family but with magnitudes reduced by 1/4.
This will produce a decompression of magnitude 1/32 arriving at the root of the tree.



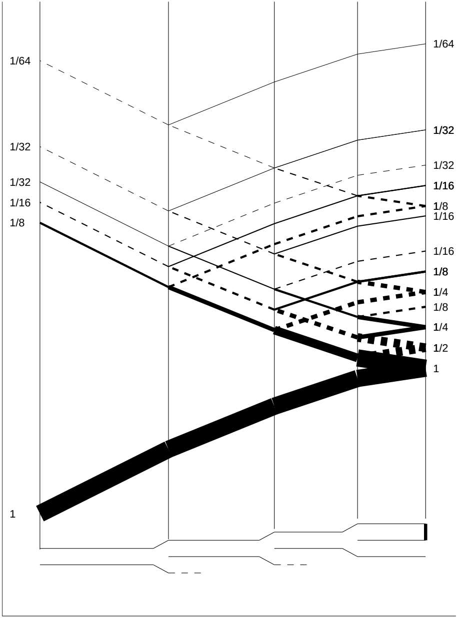
wave trapping 08

The reflection of the first reflected wavefront from the 1^{st} bifurcation, a decompression wavefront of magnitude $1/8$, will generate another family of wavefronts with magnitudes reduced by $1/8$. This will generate a decompression wavefront arriving at the root with magnitude $1/64$.



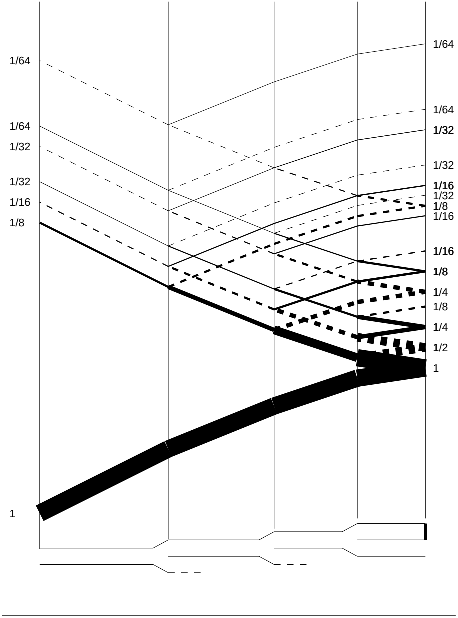
wave trapping 09

The twice reflected wavefront arriving at the occlusion as a compression wavefront with magnitude 1/4 will also generate a similar family of wavefronts to that produced by the original wavefront but with magnitudes reduced by 1/4.
This will produce a compression wavefront arriving at the root in between the first two decompression wavefronts already described. It will have magnitude 1/32.



wave trapping 10

The next compression wavefront to arrive at the root of the tree is generated by the re-reflection of the backward decompression wave from the 2nd bifurcation. It will have magnitude 1/64.
With each wavefront arriving at the occlusion producing 1 wavefront that returns to the root and 3 wavefronts that are re-reflected back to the occlusion, it is obvious that this process will generate an infinite number of wavefronts. We stopped the process when the magnitude reaches 1/128.



wave trapping 11

The complete pattern of wavefronts generated by the single compression wavefront of magnitude 1 is shown in the figure. We conclude from this simple example that the occlusion will result not in one large reflected wavefront, but in a complex sequence of ever diminishing compression and decompression wavefronts.
We also note that there are many more wavefronts in the more peripheral vessels than in the 1^{st} generation vessel. I.e. the waves are 'trapped' because of the asymmetry of the reflection coefficients at the bifurcations.