wave speed determination 01

In systemic arteries we can presume that there is a period at the start of systole when there are only forward waves in the arteries due to the initial contraction of the LV. In the coronary arteries, this initial contraction is also compressing the intramyocardial vessels and so it is not clear *a priori* when we can expect only forward or backward waves. This is an alternative way of determining ρc.

We calculate the forward and backward pressure and velocity differences from the measured pressure and velocity differences using

$$dP_{\pm} = (dP \pm \rho c dU)/2 \quad \textit{and} \quad dP_{\pm} = \pm \rho c dU_{\pm}$$

Using these we can calculate the separated wave intensities

$$dI_{\pm} \equiv dP_{\pm} dU_{\pm} = \frac{\pm(dP \pm \rho c dU)^2}{4\rho c}$$

wave speed determination 02

If we use the wrong value $\rho c' = \rho c + \epsilon$ with some additive error ϵ then at each time we will calculate the wrong value of wave intensity, $dI'_{\pm}$. Define the error term

$$\chi_n = (|dI'_+| - |dI_+|) + (|dI'_-| - |dI_-|) = (dI'_+ - dI'_-) - (dI_+ - dI_-)$$

where we have used $dI_+ \geq 0$ and $dI_- \leq 0$. Inserting from above and expanding, we obtain the error at each time

$$\begin{aligned}
\chi_n &= \frac{(dP + \rho c' dU)^2 + (dP - \rho c' dU)^2}{4\rho c'} - \frac{(dP + \rho c dU)^2 + (dP - \rho c dU)^2}{4\rho c} \\
&= \frac{dP^2 + (\rho c')^2 dU^2}{2\rho c'} - \frac{dP^2 + (\rho c)^2 dU^2}{2\rho c}
\end{aligned}$$

wave speed determination 03

In order to minimise the error over the whole cardiac cycle, define

$$\chi \equiv \sum_{n-1}^{N} \chi_n = \frac{\sum dP^2 + (\rho c')^2 \sum dU^2}{2\rho c'} - \frac{\sum dP^2 + (\rho c)^2 \sum dU^2}{2\rho c}$$

where N is the number of points in the cardiac cycle. We want to minimise this error with respect to our choice of $\rho c'$. That is

$$\frac{\partial \chi}{\partial \rho c'} = 0 = -\frac{\sum dP^2}{2(\rho c')^2} + \frac{\sum dU^2}{2}$$

This reduces to the expression for $\rho c'$ that minimises the error in the calculation of $dI_\pm$

$$(\rho c')^2 = \frac{\sum dP^2}{\sum dU^2}$$

Note that this expression is exact if it is applied to a region of the cycle when only forward or backward waves are present.

wave speed determination 04

Prof. M. Sugawara has observed that the analysis is exact only when $\Sigma dU_+ dU_- = 0$. We can show this by observing that

$$
\begin{aligned}
dP &= \rho c(dU_+ - dU_-) \\
dU &= dU_+ + dU_-
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\Sigma dP^2 &= (\rho c)^2 \Sigma(dU_+^2 + dU_-^2 - 2dU_+ dU_-) \\
\Sigma dU^2 &= \Sigma(dU_+^2 + dU_-^2 - 2dU_+ dU_-) \\
\Sigma dP dU &= (\rho c)\Sigma(dU_+^2 - dU_-^2)
\end{aligned}
$$

where Σ indicates the sum over the cardiac cycle. Combining these expressions we get expressions for the sums of the forward and backward contributions in terms of the measured pressure and velocity, assuming that ρc is known.

$$
\begin{aligned}
\Sigma dP^2 + (\rho c)^2 \Sigma dU^2 &= 2(\rho c)^2 \Sigma(dU_+^2 + dU_-^2) \\
\Sigma dU^2 - (\rho c)^2 \Sigma dU^2 &= -4(\rho c)^2 \Sigma(dU_+ dU_-)
\end{aligned}
$$

wave speed determination 05

Looking at the full expression for the ratio of the sum of the squares of pressure and velocity, we obtain

$$\frac{\Sigma dP^2}{\Sigma dU^2} = (\rho c)^2 \frac{\Sigma(dU_+^2 + dU_-^2 - 2dU_+ dU_-)}{\Sigma(dU_+^2 + dU_-^2 - 2dU_+ dU_-)}$$

If $\Sigma dU_+ dU_- = 0$ this is equal to $(\rho c)^2$.

If there is a correlation between the forward and backward waveforms, then this method will yield a inaccurate value of c.