A Graphical Generalized Implementation of SENSE Reconstruction Using Matlab

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ABSTRACT: Parallel acquisition of Magnetic Resonance Imaging (MRI) has the potential to significantly reduce the scan time. SENSE is one of the many techniques for the reconstruction of parallel MRI images. A generalized algorithm for SENSE reconstruction and theoretical background is presented. This algorithm can be used for SENSE reconstruction for any acceleration factor between 2 and 8, for any Phase Encode direction (Horizontal or Vertical), with or without Regularization. The user can select a particular type of Regularization. A GUI based implementation of the algorithm is also given. Signal-to-noise ratio, artefact power, and $g$-factor map are used to quantify the quality of reconstruction. The effects of different acceleration factors on these parameters are also discussed. The GUI based implementation of SENSE reconstruction provides an easy selection of various parameters needed for reconstruction of parallel MRI images and helps in an efficient reconstruction and analysis of the quality of reconstruction.

KEY WORDS: parallel MRI; SENSE reconstruction; regularization; k-space; Matlab

I. INTRODUCTION

Magnetic Resonance Imaging (MRI) has been of great value in the medical diagnostics for the past several years and has provided a tremendous potential to identify different pathological conditions in the human body. One major limitation of current MRI has been its long image acquisition time and as MRI equipment is expensive; it is costly to spend too much time on scanning one patient. Efforts to reduce the time of MRI scan will maximize the utility of the hospitals’ resources.

Parallel MRI is one method to reduce MRI scan time in which multiple coils (or coil arrays) are used to acquire MRI data in parallel. Many reconstruction algorithms have been suggested in the recent past that can be broadly categorized into “k-space” algorithms and “image-domain” algorithms. The work presented here is based on Sensitivity Encoding (SENSE) reconstruction suggested by Preussmann et al. (1).

SENSE is an image domain technique for reconstruction of Parallel MRI which is based on the fact that receiver sensitivity generally has an encoding
effect complementary to Fourier encoding by linear field gradients (I). Thus, the areas of the object (being imaged) which are closer to a particular coil, contribute more signal to the total signal collected by the coil as compared to the parts of the object further away from the coil. As the coils are systematically located at different parts of the object (to be imaged), their location captures spatial information in the image of the object to be reconstructed.

During image acquisition by parallel imaging, the gap between adjacent k-space lines is increased. The k-space lines are skipped to reduce acquisition time and this under sampling of k-space causes aliasing in the acquired images. As each pixel location in the aliased images has signals from more than one location of the actual image, an important step of reconstruction is to separate the signal contribution from each pixel location of the aliased image and to allocate it to the right pixel location in the reconstructed image. The sensitivity map defines the weights on the basis of which signal at each pixel location in the aliased image must be reallocated to the right pixel location in the reconstructed image. Provided that the coils' sensitivity profiles are not the same at those different locations, the weight given to each of the signal components will be different for each coil (2) thus ensuring good reconstruction.

The quality of SENSE reconstruction depends on how accurately the sensitivity map represents the weights which will be used to separate the signals in the aliased image and allocate them to the respective locations in the unwrapped image. The key to signal separation is the fact that in each single-coil image, signal superposition occurs with different weights according to their local coil sensitivities (I). Owing to its significance, proper estimation of sensitivity maps has drawn the attention of many researchers in the recent past, and many regularization techniques (3–8) have been proposed recently with the main aim of having as precise a sensitivity map as possible. Furthermore, non-Cartesian sampling methods (spiral or radial) have been recently proposed for even faster image acquisition and better navigation for flow information. Further details on this can be found in (9).

II. THEORY

The main idea in SENSE is to apply knowledge of the sensitivities of the coil elements to calculate the aliased signal component at each point (10) and then allocate these signals to their actual locations in the unfolded image. If the gap between adjacent k-space lines is increased by an acceleration factor \( R \), the signals from \( R \) locations, equally spaced along the sub-sampled direction, overlap in the image. Field of view (FOV) reduction can be stated mathematically by saying that the \( R \)-fold FOV reduction results in an \( N_A \)-fold aliased image representation as given in (1), where \( N_A \) represents the total number of signals present at location \( y \) owing to aliasing (including the actual signal of this location). Thus, for each location \( y \), we can write the image signal \( I_j(y) \) as a superposition of the original signal and displaced replicates (II):

\[
I_j(y) = \sum_{n=0}^{N_A-1} C_j(y + nL/R)M(y + nL/R)
\]

where \( j = 0, 1, \ldots, N_c - 1 \) [1]

Here \( N_c \) is the number of elements in the coil array, \( N_A \) is the number of overlapped signals at one location in the aliased image, and \( C \) stands for the encoding (or sensitivity) matrix. In the above equation, \( I_j(y) \) are known because they are the acquired aliased images (one for each coil array element). The aliased magnetization values \( M(y + nL/R) \) are to be found. If \( N_c \geq N_A \), the system of equations can be solved to obtain \( M(y + nL/R) \). The above equation can be generalized for simplicity into a matrix notation. With \( N_c \) coils, \( I \), \( C \), and \( M \) matrices can be defined with dimensions \( N_c \times 1 \), \( N_c \times N_A \), and \( N_A \times 1 \), respectively and then (1) can be written as:

\[
l = CM
\]

In SENSE, the reconstruction problem is formulated as solving a set of linear equations defined in (2) where:

\[
I = \begin{bmatrix}
I_0(y) \\
I_1(y) \\
\vdots \\
I_{N_c-1}(y)
\end{bmatrix}, \quad
M = \begin{bmatrix}
M(y) \\
M(y + L/R) \\
\vdots \\
M(y + (N_A - 1)L/R)
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
C_0(y) & \cdots & C_0(y + (N_A - 1)L/R) \\
\vdots & \ddots & \vdots \\
C_{N_c-1}(y) & \cdots & C_{N_c-1}(y + (N_A - 1)L/R)
\end{bmatrix}
\]

Here, \( I \) represents the aliased signals (from aliased images) obtained by the MRI scanner (the aliased image is obtained by skipping some phase encode lines, thus reducing the scan time), \( C \) is the encoding matrix (also named as Sensitivity Matrix) which contains spatial information about each coil.
and this information is used to relocate appropriate signals to each pixel location in the reconstructed image. "M" is the image to be recovered given by:

\[ M = C^{-1}I \]  

The inverse of matrix \( C \) in (3) can be implemented by using Moore-Penrose pseudo-inverse given by:

\[ M = \left[(C^* C)^{-1}C^*\right] I \]  

where “M” is the unfolded image.

### III. QUALITY OF RECONSTRUCTION

One issue with parallel acquisition is the loss of Signal-to-Noise Ratio (SNR) due to skipping some phase encode lines. The coil design (12–15) and the trajectory used for the k-space acquisition (16, 17) have a significant effect on the SNR with some trajectories or coil designs giving higher SNR as compared to others. Independent information from each channel in the RF coil array is very important because correlations in the spatial information from the neighboring array elements can degrade the image quality. In fact, when many receiver coils are used in conjunction with high acceleration factors, the image reconstruction may become very ill-conditioned. The standard method of reconstruction given a poorly conditioned matrix can amplify the noise in reconstructed SENSE images. The noise amplification for a poorly conditioned matrix can be reduced by a process called “regularization” (3–8). Many techniques have been proposed for regularization (18–23), and these regularization techniques use different ways to decrease the ill-conditioning of the sensitivity maps.

If we examine the image from a single element of the array, the signal values are generally confined to a region near the coil element, but the noise values are distributed throughout the image. If the noise from the different elements is weakly correlated (or incoherent), the noise in the reconstructed image will grow as the square root of the number of elements while the signal will grow as the number of elements, provided the signal phases are aligned and all the coils are equally sensitive (24).

The acceleration factor gives the number of times the data from each coil is reused to calculate the final unwrapped image. When this data is reused, the noise is necessarily amplified because it has natural correlation with itself. So, when a particular point is weakly detected by several coil elements or well detected by only a single element, the composite image will see higher noise due to this autocorrelation effect (24). The quantification of this noise amplification factor is done with “g-factor.” The “g-factor” describes how well the coil array encodes the magnetization distribution of the object. A smaller g-factor generally indicates that the magnetization at a given location in the object is detected by several coil elements. Provided the noise correlation between those elements is weak, greater sensitivity can be recovered than if only a single element of the array can detect the magnetization (24). The relationship between the SNR with and without SENSE is given by (25)

\[ \text{SNR}_{\text{SENSE}} = \frac{\text{SNR}_{\text{Normal}}}{g \sqrt{R}} \]  

Here, the factor “R” implies the expected loss in SNR that results by reducing the scan time by acceleration factor “R” and “g” is the geometry factor which represents noise magnification that occurs when aliasing is reconstructed. The g-factor is determined by (1):

\[ g_i = \sqrt{\left|\left(C \Psi^{-1} C\right)^{-1}\right|_{ii}} \]  

Here “\( \Psi \)” is the \( N_c \times N_c \) noise correlation matrix for the coils in which a diagonal element represents noise variance from a single coil and an off-diagonal element represents a noise cross-correlation between two coils and “C” is the encoding matrix. This equation applies to all pixels in the image with the same number of aliased replicates, i.e., \( N_A \). The subscript “i” refers to aliased replicate number “i” for that pixel and has the range 0,1,....,\( N_A - 1 \). Thus, the geometry factor for all pixels that are related by aliasing at a particular location in the aliased image can be computed by the above equation. The noise amplification described by the g-factor is also related to a property of the matrix “\( C^* \Psi^{-1} C \)” that is inverted in (6), called its conditioning (3–8).

The g-factor is a measure of correlation between the neighboring coils and indicates the noise magnification capability of a coil array and it depends on the number of aliased replicates \( N_A \) as well as on the coil sensitivity difference between aliased pixels. The sensitivity difference depends on the coil conductor placement, the scan plane orientation, the phase encode direction within the scan plane, and the pixel location within the scan plane. Therefore, g-factor is quite useful when considering how to design a coil which is to be used for SENSE.
IV. IMPLEMENTATION

The SENSE reconstruction (4) has been implemented using a GUI interface that allows the user to define all the key variables for the reconstruction, as shown in Fig. 1. The user defined variables for this application are: (1) aliased images, (2) sensitivity map, (3) original image (for comparison), (4) phase encode direction, i.e., horizontal or vertical, (5) acceleration factor between two and eight, (6) regularization type: a: Polynomial (order 1), b: Polynomial (order 2), c: Tikhonov Regularization, d: Wavelet based regularization, (7) Under-sampling: User can select if the loaded data is already aliased and there is no need for under-sampling or loaded data needs to be under-sampled to simulate aliased data. Output: (1) reconstructed image, (2) g-factor map, (3) SNR value, (4) artefact power. The flowchart for this generalized SENSE reconstruction is given in Fig. 2. For ease of use, this algorithm has been linked with a GUI interface. Matlab (version 7.6) has been used for this GUI platform and all the functions needed during reconstruction have been associated with different fields in the GUI for an efficient working.

In MRI, the raw data we acquire are complex (having a magnitude and a phase component). Normally, the magnitude is enough to visualize an MRI image and the phase component is ignored. However, phase plays a crucial role in the SENSE reconstruction process. The coil sensitivity varies in space in both phase and magnitude and the phase information is essential for an accurate reconstruction. So, all the steps shown in the algorithm involve complex arithmetic.

V. EVALUATION OF RECONSTRUCTION

The performance of the parallel image reconstruction algorithm can be evaluated by two quantification parameters: (1) signal to noise ratio (SNR) (2) artefact power

1. Signal to Noise Ratio (SNR): During the process of reconstruction, the user is asked to select a region of interest for signal (ROS) and a region of interest for noise (RON), normally the background. Then SNR is calculated by using the following formula (26):

\[
\text{SNR}(\text{dB}) = 20 \log_{10} \frac{\text{Mean ROS}}{\text{Std.Deviation of RON}} \quad [7]
\]

2. Artefact Power (AP): The concept of AP has been derived from “Square Difference Error.” Here, it is presumed that a reference image (full FOV) is available and the AP in the reconstructed image will be evaluated on the
basis of this reference image. AP can be calculated using the following formula (26):

$$AP = \frac{\sum |{reference(x, y)}| - |{reconstructed(x, y)}|^2}{\sum |{reference(x, y)}|^2}$$ \[8\]

It is clear from the above formula that if \(I_{\text{reference}} = I_{\text{reconstructed}}\), the AP will be zero meaning that there is no artefact in the reconstructed image and the reconstructed image is identical to the reference image. Similarly, AP will be a bigger value (i.e., closer to 1) if the reconstructed image is significantly different than the reference image.

### VI. RESULTS AND DISCUSSION

To demonstrate the performance of this implementation, the reconstruction algorithm is first applied on simulated data and then on the experimental data sets. We used a 1.5 Tesla GE scanner at St. Mary’s Hospital London with an eight channel head coil and
a Gradient Echo sequence with the following parameters: TE = 10 m sec, TR = 500 m sec, FOV = 20 cm, Bandwidth = 31.25 KHz, Slice Thickness = 3 mm, Flip Angle = 90°, Matrix Size = 256 × 256.

For both datasets, the full k-space data were acquired (Fig. 3) and their “sum of squares” reconstruction was used as a reference image. Then, the specified number of k-space lines were skipped to produce aliased images, depending upon the acceleration factor, e.g., for an acceleration factor of two, one out of every two phase encode steps were removed, for an acceleration factor of three, two out of every three phase encode steps were removed. The inverse Fourier transform of the sub-sampled k-space gave us aliased images. To have the sensitivity map, the central lines of k-space of the full FOV data were truncated by using cosine taper window (25). In this way, low resolution images of the coils were obtained and then these values were normalized by dividing by sum of square image, thus giving us the sensitivity maps (Fig. 4). Information about the noise captured by the coils during imaging process is very useful in the reconstruction process. The noise images are obtained by switching off the RF signal and capturing the images. Thus, the signals obtained are just the noise images and not the signals from the

Figure 3 Images acquired by eight separate coils of an eight array head coil showing differing spatial localization of the signals.

Figure 4 Sensitivity maps obtained by dividing low resolution coil images (of Fig. 3) by sum of squares image.
object being imaged. These aliased images (Fig. 5) along with sensitivity information (Fig. 4) are used to reconstruct the image [Fig. 6(b)].

Figure 7 indicates the relationship between acceleration factor and SNR, acceleration factor and artefact power. The reconstruction was performed using the above algorithm for various acceleration factors ranging between 2 and 8. It is noticed that the SNR deteriorates abruptly after acceleration factor of 4. Similarly, there is a sharp increase in the artefact power after acceleration factor of 5. The possible reason for this may be the fact that the receiver coils must have independent and distinct sensitivity profiles to give a good reconstruction. It means that we have to see how many distinct receiver channels are there in the direction of phase encoding. It is quite probable that some of the eight receiver coil channels may not have distinct sensitivity profiles particularly in the phase encoding direction. Although, theoretically we say that the maximum achievable acceleration factor from a receiver coil array is less than maximum number of coils present in the array, but it is valid only if each coil has a distinct profile. If this condition is not fulfilled then reconstruction will not be of good quality for higher acceleration factors.

Another important point to consider is that the coil array system used here is circular. A circular array has sensitivity variations not only in the direction of phase encoding but along both x and y axis. It gives a complex profile of coil sensitivity as compared to a Linear Array (in which maximum sensitivity change is only in one direction either x or y). This two directional change in sensitivity may add some inconsistency to the reconstruction because it becomes difficult to have accurate estimation of the sensitivity changes in both directions.

These results validate the accuracy of the reconstruction algorithm. It is important to note that these results are obtained without any regularization. Better results may be obtained with regularization and
better coil design especially for higher acceleration factors because these measures can help improve the SNR as well as decrease the artefact power. This will be the subject of our future work.

VII. CONCLUSIONS

A detailed account of a GUI based implementation of SENSE reconstruction is presented. There are several parameters to be defined during the process of image reconstruction, e.g., phase encode direction (horizontal or vertical), acceleration factor, regularization type etc. This algorithm presents a generalized SENSE reconstruction method giving a flexibility to select different parameters needed for the reconstruction. The GUI interface provides a more manageable way to load the aliased images, sensitivity maps and other related data as well as facilitates the selection of different reconstruction parameters in an interactive way. The results of reconstruction and the $g$-factor map are also displayed on the same window thus making it easy to analyze the reconstruction. SNR, $g$-factor, and artefact power are the parameters which are used to quantify the quality of reconstruction. The user can under-sample the k-space, if there is a need to produce aliased images. It is noticed that the $g$-factor deteriorates badly as soon as the acceleration factor exceeds 4 (Fig. 7). This is due to a significant loss in the k-space data because for an acceleration factor of 5, just one out of five phase encode lines are acquired thus causing a loss in SNR. Higher acceleration factors result in more phase encode steps being skipped causing even more degradation in SNR. This tool is available freely on request to the corresponding author.

REFERENCES


BIOGRAPHIES

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